MARKING SCHEME MATHEMATICS MODEL PAPER CLASS 10

SECTION –A

Time: 20 Minutes **Marks:** 15

Scoring Keys:

1. The quadratic equation in the following is:

A. $x^4 + 11x^2 + 9 = 0$

B. $x^3 + 11x^2 + 9 = 0$

C. $x^3 + 11x + 9 = 0$

D. $\boldsymbol{x^2 + 11x + 9 = 0}$

2. The solution set of $2x^2 - 9x + 5 = 0$ is:

A. $\left\{\frac{-9\pm\sqrt{41}}{4}\right\}$

B. $\boldsymbol{\left\{\frac{9\pm\sqrt{41}}{4}\right\}}$

C. $\left\{\frac{-9\pm\sqrt{41}}{2}\right\}$

D. $\left\{\frac{-9\pm\sqrt{41}}{2}\right\}$

3. $\frac{1}{\alpha} + \frac{1}{\beta} =$

A. $\frac{1}{\alpha\beta}$

B. $\frac{1}{\alpha+\beta}$

C. $\frac{\alpha\beta}{\alpha+\beta}$

D. $\boldsymbol{\frac{\alpha+\beta}{\alpha\beta}}$

4. The discriminant of equation $x^2 + 6x + 2 = 0$ is equal to:

A. 8

B. 28

C. 36

D. 44

5. Direct variation between p and q can be expressed as:

A. $p = q$

B. $p = \frac{1}{q}$

C. $\boldsymbol{p \propto q}$

D. $p \propto \frac{1}{q}$

6. In continued proportion $p:q = q:r, r$ is called as:

A. first proportional to p, q.

B. second proportional to p, q.

C. third proportional to $\boldsymbol{p}, \boldsymbol{q}$.

D. fourth proportional to p, q.

7. $\frac{x^2+1}{x+1}$ is an example of:

A. proper fraction only

B. improper fraction only

C. both proper and rational fraction

D. both improper and irrational fraction

8. The set of the whole numbers (W) in the following is:

A. $\{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \dots \dots \dots\}$

B. $\{0, \pm 2, \pm 4, \dots \dots \dots\}$

C. $\{1,2,3, \dots \dots \dots\}$

D. $\{0, \pm 1, \pm 2, \pm 3, \dots \dots \dots\}$

9. The range of $R = \{(1,2), (2,2), (3,1), (4,4)\}$ is:

A. $\{1,3,4\}$

B. $\{\mathbf{1}, \mathbf{2}, \mathbf{4}\}$

C. $\{2,3,4\}$

D. $\{1,2,3,4\}$

10. If $A = \{1,2,3,4\}$ and $B = \{5,6,7,8\}$, then which of the following binary relations is a function from B to A?

A. $R = \{(1,5), (2,6), (3,7), (4,8)\}$

B. $R = \{(1,6), (2,5), (4,8), (4,7)\}$

C. $\boldsymbol{R} = \{(\mathbf{5}, \mathbf{1}), (\mathbf{6}, \mathbf{2}), (\mathbf{7}, \mathbf{3}), (\mathbf{8}, \mathbf{4})\}$

D. $R = \{(5,2), (6,1), (8,4), (8,3)\}$

11. The value that appears more times in a data is called:

A. mean

B. median

C. mode

D. variance

12. In the given set of data, $71, 73, 79, 77, 76, 75, 80$, the median is:

A. 73

B. 76

C. 77

D. 79

13. In radians, $45°$ is equal to:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\boldsymbol{\frac{\pi}{4}}$

D. $\frac{\pi}{6}$

14. $1 + cot^2\theta =$

A. $sin^2\theta$

B. $cos^2\theta$

C. $tan^2\theta$

D. $\boldsymbol{cosec^2\theta}$

15. The number of circles that can pass through three non-collinear points is:

A. 0

B. $\mathbf{1}$

C. 2

D. 3

KEY:

MCQs No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Key	**D**	**B**	**D**	**B**	**C**	**C**	**C**	**A**	**B**	**C**	**C**	**B**	**C**	**D**	**B**

SECTION-B

Time: 2 Hours 40 Minutes **Marks:** 36

1. Attempt any **NINE** of the following short questions. Each question carries 4 marks.

i. Derive quadratic formula for $ax^2 + bx + c = 0$ where $a \neq 0$, by using completing square method.

Solution:

As general form of quadratic equation is

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$\div$ *ing by "a"*

$$x^2 + 2(x)\left(\frac{b}{2a}\right) = \frac{-c}{a}$$

Step-1 (1 Mark)

Adding $\left(\frac{b}{2a}\right)^2$ *on B.S*

$$x^2 + 2(x)\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

using $a^2 + 2ab + b^2 = (a+b)^2$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step-2 (1 Mark)

Taking Square root on B.S

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Step-3 (1 Mark)

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which is a required quadratic formula.

Step-4 (1 Mark)

ii. Solve $4.2^{2x} - 10.2^x + 4 = 0$.

Solution:

$$4.2^{2x} - 10.2^x + 4 = 0$$

We can write:

$4. (2^x)^2 - 10.2^x + 4 = 0 \longrightarrow$ ❶

$Let\ y = 2^x \longrightarrow$ ❷

❶ $\Rightarrow 4(y)^2 - 10y + 4 = 0$

$4y^2 - 10y + 4 = 0$

Step-1

$4y^2 - 8y - 2y + 4 = 0$

$4y(y-2) - 2(y-2) = 0$

$(y-2)(4y-2) = 0$

So either $y - 2 = 0$ *or* $4y - 2 = 0$

$y = 2$ $\quad$ $4y = 2$

$\frac{4}{4}y = \frac{2}{4}$

$y = \frac{1}{2}$

Step-2

Re-putting Values:

$y = 2^x$

$2^x = 2^1$

By Comparing:

$x = 1$

Step-3

$2^x = \frac{1}{2}$

$2^x = 2^{-1}$

$x = -1$

Step-4

$Solution. Set = \{1, -1\}.$

iii. Find the cube roots of 64.

Solution:

$Let\ x\ be\ the\ cube\ root\ of\ 64\ i.e.$

$x = \sqrt[3]{64}$

$x = 64^{1/3}$

$Taking\ Cube\ on\ B.S$

$x^3 = 64^{3\times 1/3}$

$x^3 - 64 = 0$

$x^3 - 4^3 = 0$

$(x-4)(x^2 + 4x + 16) = 0$ $\quad By\ using$ $\quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Step-1

So either

$x^2 + 4x + 16 = 0$ *or* $x - 4 = 0$

$Here\ a = 1, b = 4, c = 16 \quad or \qquad x = 4$

So $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Step-2

$= \frac{-4 \pm \sqrt{4^2-4(1)(6)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16-64}}{2}$

$= \frac{-4 \pm \sqrt{-48}}{2}$

$= \frac{-4 \pm 4i\sqrt{3}}{2}$

$= 4(\frac{-1 \pm i\sqrt{3}}{2})$

Step-3

$x = 4\left(\frac{-1+i\sqrt{3}}{2}\right), x = 4\left(\frac{-1-i\sqrt{3}}{2}\right)$

$x = 4w, x = 4w^2$

$Hence\ \{4, 4w, 4w^2\}\ are\ cube\ roots\ of\ 64.$

Step-4

iv. If α, β are roots of $x^2 - 4x + 2 = 0$, find the equation whose roots are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$.

Solution:

$As\ for\ x^2 - 4x + 2 = 0, \quad \alpha + \beta = -\frac{b}{a} = -\frac{-4}{1} = 4$ and $\alpha\beta = \frac{c}{a} = \frac{2}{1} = 2$

Required equation whose roots are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ $is\ x^2 - Sx + P = 0 \longrightarrow$ ❶

Step-1

$For\ S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ & $P = \frac{\alpha}{\beta} \times \frac{\beta}{\alpha}$

$S = \frac{\alpha^2+\beta^2}{\alpha\beta}$ & $P = 1$

Step-3

$S = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$S = \frac{4^2-2(2)}{2} \Rightarrow \frac{16-4}{2} \Rightarrow \frac{12}{2}$

$S = 6$

Step-2

$Put\ S = 6\ and\ P = 1\ in$ ❶

So $\Rightarrow x^2 - 6x + 1 = 0\ is\ a\ required\ solution.$

Step-4

v. Find the mean proportional of $a^2 - b^2$ and $\frac{a+b}{a-b}$.

Solution:

Let x be the mean proportional of $a^2 - b^2$ and $\frac{a+b}{a-b}$.
i.e. $a^2 - b^2 : x : \frac{a+b}{a-b}$ are in continued proportion.
$\Rightarrow a^2 - b^2 : x :: x : \frac{a+b}{a-b}$
$As\ product\ of\ mean = product\ of\ extreme$

Step-1

$x^2 = a^2 - b^2 \times \frac{a+b}{a-b}$ } Step-2 $x^2 = (a+b)(a-b) \times \frac{a+b}{a-b}$ $x^2 = (a+b)^2$ } Step-3 Taking square root on B.S. $\sqrt{x^2} = \sqrt{(a+b)^2}$ $x = \pm(a+b)$ } Step-4
vi. Resolve into partial fraction $\frac{4x+2}{(x+2)(2x-1)}$. **Solution**: $Let\ \frac{4x+2}{(x+2)(2x-1)} = \frac{A}{x+2} + \frac{B}{2x-1} \longrightarrow$ ❶ $\times\ ing\ B.S\ by\ (x+2)(2x-1)$ $4x+2 = A(2x-1) + B(x+2) \longrightarrow$ ❷ } Step-1 $Put\ 2x-1 \Rightarrow x = \frac{1}{2}\ in$ ❷ $4(\frac{1}{2}) + 2 = A\left(2(\frac{1}{2}) - 1\right) + B(\frac{1}{2} + 2)$ $4 = B(\frac{5}{2})$ $B = \frac{8}{5}$ } Step-2 $Put\ x+2 = 0 \Rightarrow x = -2$ in ❷ $4(-2) + 2 = A(2(-2) - 1) + B(-2+2)$ $-8 + 2 = A(-4-1)$ $-6 = A(-5)$ $A = \frac{6}{5}$ } Step-3 $Put\ A = \frac{6}{5}\ and\ B = \frac{8}{5}\ in$ ❶ $\frac{4x+2}{(x+2)(2x-1)} = \frac{\frac{6}{5}}{x+2} + \frac{\frac{8}{5}}{2x-1}$ $= \frac{6}{5(x+2)} + \frac{8}{5(2x-1)}\quad Ans.$ } Step-4
vii. If $U = \{1,2,3, \dots \dots ,10\}$, $A = \{2,4,6,8,10\}$ and $B = \{1,3,5,7,9\}$, then verify $(A \cup B)' = A' \cap B'$. **Solution**: $Here, U = \{1,2,3, \dots \dots ,10\}$, $A = \{2,4,6,8,10\}$ and $B = \{1,3,5,7,9\}$ $To\ Prove: (A \cup B)' = A' \cap B'$. First: $A \cup B = \{2,4,6,8,10\} \cup \{1,3,5,7,9\}$ $A \cup B = \{1,2,3,4,5,6,7,8,9,10\}$ } Step-1 $\boldsymbol{L.H.S}: (A \cup B)'$ $U - (A \cup B)' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5,6,7,8,9,10\}$ $= \{\ \} \longrightarrow$ ❶ } Step-2 & $A' = U - A = \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$ $A' = \{1,3,5,7,9\}$ & $B' = U - B = \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}$ $B' = \{2,4,6,8,10\}$ } **Step-3** $\boldsymbol{R.H.S} = A' \cap B'$ $A' \cap B' = \{1,3,5,7,9\} \cap \{2,4,6,8,10\}$ $= \{\ \} \longrightarrow$ ❷ **From** ❶ & ❷ $\boldsymbol{L.H.S} = \boldsymbol{R.H.S}$ i.e. $(A \cup B)' = A' \cap B'$. } **Step-4**
viii. A set of data contains the values as $105, 80, 90, 75, 100, 105\ and\ 110$. Show

that $Mode > Median > Mean$.

Proof:

Given Data is $105,80,90,75,100,105$ and 110.

To Find Mean:

$Mean = \frac{x_1+x_2+x_3+x_4+x_5+x_6+x_7}{7}$

$Mean = \frac{105+80+90+75+100+105+110}{7}$ Step-1

$Mean = 95 \rightarrow$ ❶

To Find Median: First arrange data in ascending order i.e.

$75,80,90,100,105,105,110$ Step-2

$Median = 100 \rightarrow$ ❷

To Find Mode: Mode is the most frequent value, So.

$Mode = 105 \rightarrow$ ❸ Step-3

From ❶,❷ and ❸,

$Mode > Median > Mean$ Step-4

$105 > 100 > 95$

ix. An arc of a circle subtends an angle of 2 radians at the center. If the area of sector formed is $64cm^2$, find the radius of the circle.

Solution:

$\theta = 2\,radian$

$Area = 64\ cm^2$

$r = ?$

We know that: Step-1

$A = \frac{1}{2}r^2\theta$

$64 = \frac{1}{2}r^2(2)$ Step-2

$64 = r^2 \Rightarrow r^2 = 64$ Step-3

Taking square on B.S

$r = 8cm$ Step-4

x. Prove that: $cosx - cosx\ sin^2x = cos^3x$.

Proof:

$\boldsymbol{L.H.S} = cosx - cosx\ sin^2x$

$cosx(1 - sin^2x) = 1$ } Step-1

$As \quad \cos^2\theta + \sin^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta$ } Step-2

$= cosx(\cos^2 x)$ } Step-3

$= \cos^3 x$

$= \boldsymbol{R.H.S}$ } **Step-4**

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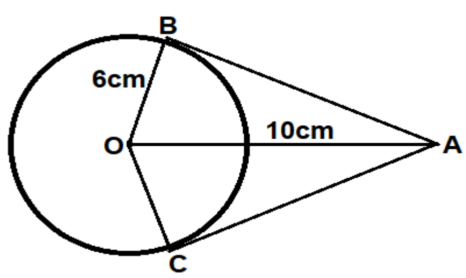
xi. $\overline{AB}$ and $\overline{AC}$ are tangent segments to the circle with centre O. If $m\overline{OB} = 6cm$ and $m\overline{OA} = 10cm$, then find $m\overline{AB}$ and $m\overline{AC}$.

Solution:



Step-1

Since AOB is a right triangle.

$\therefore \ (m\overline{OA})^2 = (m\overline{AB})^2 + (m\overline{OB})^2$ } Step-2

$(10)^2 = (m\overline{AB})^2 + (6)^2$

$100 = (m\overline{AB})^2 + 36$

$(m\overline{AB})^2 = 100 - 36$

$(m\overline{AB})^2 = 64$ } Step-3

$m\overline{AB} = 8cm$

$m\overline{AB} = m\overline{AC} = 8cm$ } Step-4

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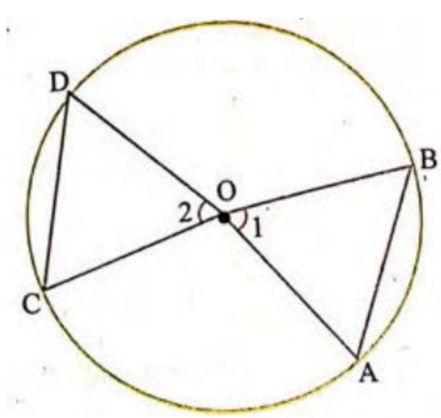
i. Prove that equal chords of a circle subtend equal angles at the center. Prove for only one circle.

Proof:

Given:



A circle with center O. $\overline{AB}$ and $\overline{CD}$ are two chords of the circle (which are not diameters) such that $\overline{AB} \cong \overline{CD}$ or $m\overline{AB} \cong m\overline{CD}$.

Arcs subtend $\angle 1$ and $\angle 2$ at the center.

To Prove: $\angle 1 = \angle 2$

Construction: We Join O to A, B, C and D respectively so that $m\overline{OA} =$

$m\overline{OB} = m\overline{OC} = m\overline{OD} = radii$ of a circle.

Proof:

Statements	Reasons
$In\ \Delta OAB \leftrightarrow \Delta OCD$	
$\overline{OA} \cong \overline{OC}$	Radii of same circle.
$\overline{OB} \cong \overline{OD}$	Radii of same circle.
$\overline{AB} \cong \overline{CD}$	Given
$\therefore$ $\Delta OAB \cong \Delta OCD$	$S.S.S \cong S.S.S$
$\therefore$ $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles.

Given & To Prove	Construction	Proof
01 Mark	**01 Mark**	**02 Marks**

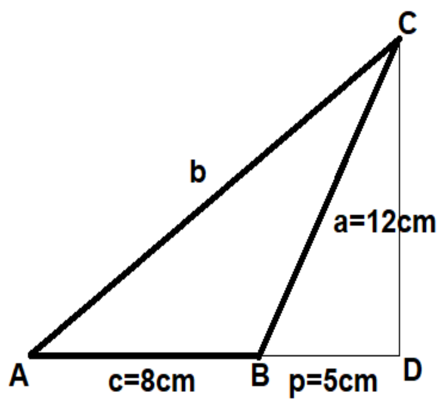
SECTION-C

Marks: 24

NOTE: Attempt any **THREE** of the following questions. Each question carries 8 marks.

2. In $\Delta ABC, m\overline{AB} = 8cm, m\overline{BC} = 12cm,\ m\angle B = 100^{o}$. The projection of $\overline{BC}$ on $\overline{AB}$ is $6cm$. Find $m\overline{AC}$.

Solution:



Step-1 (04 Marks)

Since,

$b^2 = a^2 + c^2 + 2cp$ — Step-2 (01 Mark)

$\therefore\ b^2 = (12)^2 + (8)^2 + 2(8)(5)$

$b^2 = 144 + 64 + 80$ — Step-3 (01 Mark)

$b^2 = 288$ — Step-4 (01 Mark)

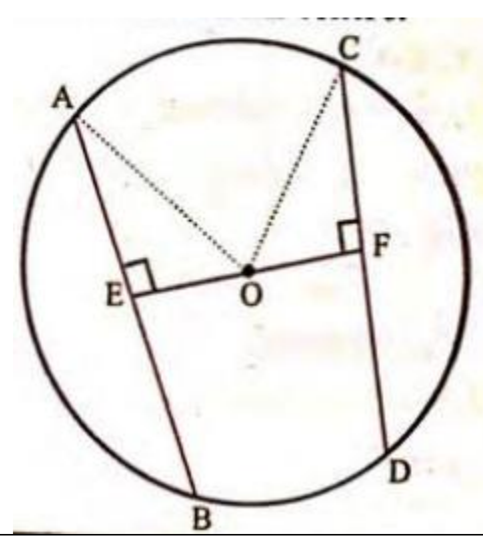
$b = 16.97cm$ — Step-5 (01 Mark)

3. Prove that If two chords of a circle are congruent then they will be equidistant from the center.

Given:

A circle with center O, $\overline{AB}$ and $\overline{CD}$ are two congruent chords of the circle.

To Prove: $\overline{AB}$ and $\overline{CD}$ are equidistant from the center O.



Construction: Join O to A and C. Also draw perpendicular $\overline{OE}$ and $\overline{OF}$ on the given chords $\overline{AB}$ and $\overline{CD}$ respectively.

Proof:

Statements	Reasons
$Since\ \overline{OE} \perp \overline{AB}\ and\ \overline{OF} \perp \overline{CD}$	Construction
$\therefore \overline{AE} \perp \overline{EB}\ and\ \overline{CF} \perp \overline{DF}$	By the use of Theorem 9.3
$or\quad m\overline{AE} = m\overline{EB}\ and\ m\overline{CF} = m\overline{DF}$	

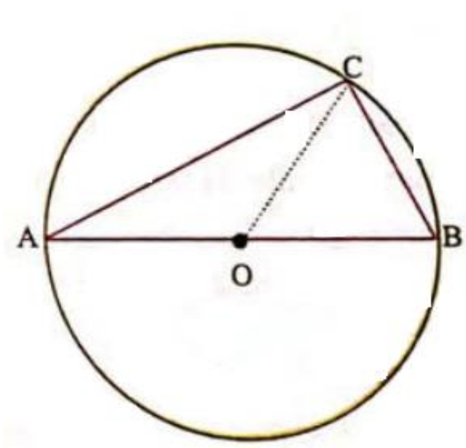
But $m\overline{AB} \cong m\overline{CD}$	Given
or $m\overline{AE} + m\overline{EB} = m\overline{CF} + m\overline{DF}$	Segment addition postulate
$m\overline{AE} + m\overline{AE} = m\overline{CF} + m\overline{CF}$	$\because m\overline{EB} = m\overline{AE}$ and $m\overline{DF} = m\overline{CF}$
$2m\overline{AE} = 2m\overline{CF}$	Adding equal quantities.
$m\overline{AE} = m\overline{CF}$	Dividing both sides by 2.
Or $\overline{AE} = \overline{CF} \rightarrow$ ❶	
Now, in $\Delta AOE \leftrightarrow \Delta COF$	
$\overline{OA} = \overline{OC}$	Radii of the same circle
$\overline{AE} = \overline{CF} \rightarrow$ ❷	From ❶ proved above
$\angle AEO \cong \angle CFO$	Right angles
$\therefore \Delta AOE \cong \Delta COF$	$H.S \cong H.S$
$\therefore \overline{OE} = \overline{OF}$ or $m\overline{OE} = m\overline{OF}$	Corresponding sides of the triangle.
$\therefore \overline{AB}$ and $\overline{CD}$ are equidistant from the center of the circle.	

Given	To Prove	Construction	Proof
01 Mark	**01 Mark**	**02 Marks**	**04 Marks**

4. Prove that the angle in a semi-circle is a right angle.

Given:

A circle with center O, $\overline{AB}$ is a diameter of the circle and $\angle ACB$ is the any angle in the semi-circle.



To Prove: $\angle ACB$ is a right angle i.e. $m\angle ACB = 90°$.

Construction: Join O to A and C.

Proof:

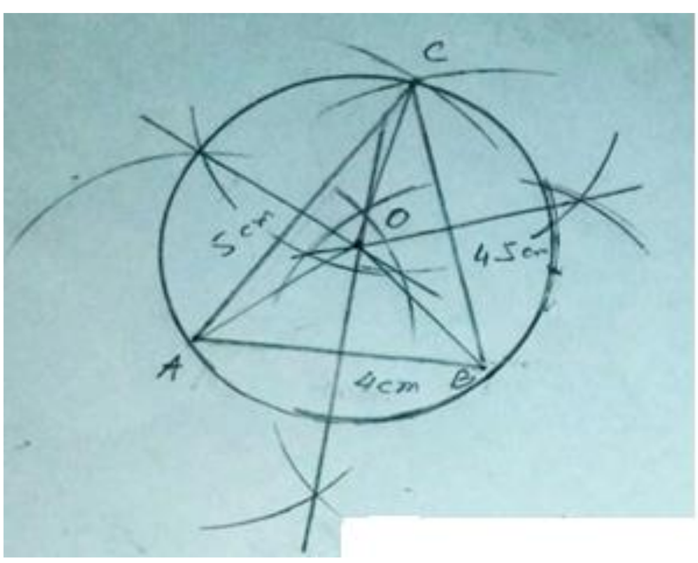
Statements	Reasons
$In\ \Delta OAC,$	
$m\overline{OA} \cong m\overline{OC}$	Radii of the same circle.

	$\therefore \Delta OAC$ *is an isoceles triangle.*	Definition of *isoceles triangle*
	and $m\angle OAC \cong m\angle OCA \rightarrow$ ❶	If two sides of a triangle are equal, the angles which are opposite to them are also equal.
	Similarly in the ΔOCB	
	$m\overline{OB} \cong m\overline{OC}$	Radii of a circle.
$\therefore$	*and* $m\angle OBC \cong m\angle OCB \rightarrow$ ❷	
$\therefore$	$m\angle OAC + m\angle OBC = m\angle OCA + m\angle OCB$	Adding ❶ and ❷
	$m\angle OAC + m\angle OBC = m\angle ACB \rightarrow$ ❸	$\because m\angle OCA + m\angle OCB = m\angle ACB$
	But $m\angle OAC + m\angle OBC + m\angle ACB = 180°$	$\because$ *The sum of three angles of a triangle is equal to* $180°$.
	Or $m\angle ACB + m\angle ACB = 180°$	$\because m\angle OAC + m\angle OBC = m\angle ACB$ Adding two equal numbers.
	$\Rightarrow m\angle ACB = 90°$	
or	$\angle ACB$ *is a right angle.*	

The angle inscribed in a semicircle is always a right angle$(90°)$.

Given	To Prove	Construction	Proof
01 Mark	**01 Mark**	**02 Marks**	**04 Marks**

5. Construct a triangle with sides 4 cm, 4.5 cm and 5 cm. Also draws its circumcircle.



Steps of Construction:

i. Draw a line segment $\overline{AB} = 4cm$.
ii. At point "B" draw an arc of radius 4.5cm.
iii. At point "A" draw an arc of radius 5cm.
iv. Both arcs intersect at point C.
v. Join A, B to C.
vi. ΔABC is a required triangle.
vii. Draw right bisector of $\overline{AB}, \overline{BC} and\ \overline{CA}$.
viii. All right bisector can pass through O.
ix. Draw radius $\overline{OA}, \overline{OB} and\ \overline{OC}$.

x. Draw a circle of radius $\overline{OA}, \overline{OB}$ *or* $\overline{OC}$, which is the required circumcircle of the given triangle.

Construction	**Steps of Construction**
04 Marks	04 Marks